Online Supplement for "A Note on "A LP-based Heuristic for a Time-Constrained Routing Problem""

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The master problem (MP) for this instance of ITP with 20 sites ($i = 0, \ldots, 19$) and 100 tours ($j = 0, \ldots, 99$) over a planning horizon of 5 days ($t = 0, \ldots, 4$) is given in `Avella.lp` `(Avella.mps)`[1] with an optimal objective function value of 974.04.

The restricted master problem (RMP) given in `AvellaRMP.lp (AvellaRMP.mps)` is initialized with 21 tours. The associated optimal objective function value is 972.60, and the optimal dual values are obtained as:

$$\begin{array}{lllll}
\pi_0 = 47.4, & \pi_1 = 33.6, & \pi_2 = 69.4, & \pi_3 = 0.0, & \pi_4 = 0.0, \\
\pi_5 = 26.0, & \pi_6 = 40.8, & \pi_7 = 42.4, & \pi_8 = 44.2, & \pi_9 = 48.4, \\
\pi_{10} = 0.0, & \pi_{11} = 78.8, & \pi_{12} = 11.0, & \pi_{13} = 65.8, & \pi_{14} = 38.4, \\
\pi_{15} = 29.2, & \pi_{16} = 98.6, & \pi_{17} = 59.2, & \pi_{18} = 26.4, & \pi_{19} = 0.0, \\
\lambda_0 = 42.6, & \lambda_1 = 42.6, & \lambda_2 = 42.6, & \lambda_3 = 42.6, & \lambda_4 = 42.6.
\end{array}$$

Using these dual variables, $\bar{r}_j = r_j - \sum_{i:j \in D(i)} \pi_i - \sum_{t:j \in F(t)} \lambda_t \leq 0$, for each $j \in J$, and the column-and-row generation algorithm in Avella et al. (2006) terminates with a suboptimal solution. The correct stopping condition for a column-and-row generation algorithm that solves the LP relaxation of ITP is specified in Theorem 1.2 in our paper and requires that

$$\bar{c}_j = r_j - \sum_{i:j \in D(i)} \pi_i - \min_{t \in S(j)} \lambda_t \leq 0, \qquad j \in J \setminus \bar{J}. \tag{1}$$

Iteration 1. Tour 44 covers sites 2, 10, 15, and 19, and may be performed on days 1 and 2. Based on (1), the reduced cost of y_{44} is calculated as below in our algorithm:

$$\bar{c}_{44} = 152.0 - (\pi_2 + \pi_{10} + \pi_{15} + \pi_{19}) - \min\{\lambda_1, \lambda_2\} = 10.8 > 0.$$

Adding y_{44} to the RMP, we obtain `AvellaRMP1.lp`. The optimal objective function value associated with this RMP is 972.60, and we obtain the following dual variable values:

$$\begin{array}{lllll}
\pi_0 = 31.2, & \pi_1 = 17.4, & \pi_2 = 64.0, & \pi_3 = 0.0, & \pi_4 = 0.0, \\
\pi_5 = 4.4, & \pi_6 = 30.0, & \pi_7 = 37.0, & \pi_8 = 55.0, & \pi_9 = 43.0, \\
\pi_{10} = 0.0, & \pi_{11} = 84.2, & \pi_{12} = 0.2, & \pi_{13} = 55.0, & \pi_{14} = 27.6, \\
\pi_{15} = 18.4, & \pi_{16} = 98.6, & \pi_{17} = 48.4, & \pi_{18} = 10.2, & \pi_{19} = 0.0, \\
\lambda_0 = 69.6, & \lambda_1 = 69.6, & \lambda_2 = 69.6, & \lambda_3 = 69.6, & \lambda_4 = 69.6.
\end{array}$$

[1] All .lp and .mps files reside in the same folder as this document. The `IBM ILOG CPLEX 12.1 Interactive Optimizer` was invoked on these files to obtain the results in this supplement.

Iteration 2. Tour 31 covers sites 1, 6, and 13, and may be scheduled on days 0 and 4. Using the dual values above, our algorithm determines that the reduced cost of column y_{31} is positive:

$$\bar{c}_{31} = 184.0 - (\pi_1 + \pi_6 + \pi_{13}) - \min\{\lambda_0, \lambda_4\} = 12.0.$$

After adding y_{31}, the resulting RMP given in `AvellaRMP2.lp` is solved. The objective function value increases to 972.70, and the corresponding optimal dual solution is determined as:

$$\begin{array}{lllll}
\pi_0 = 36.8, & \pi_1 = 22.7, & \pi_2 = 69.8, & \pi_3 = 0.0, & \pi_4 = 0.0, \\
\pi_5 = 15.5, & \pi_6 = 30.0, & \pi_7 = 30.8, & \pi_8 = 43.4, & \pi_9 = 36.8, \\
\pi_{10} = 0.0, & \pi_{11} = 90.2, & \pi_{12} = 0.0, & \pi_{13} = 66.6, & \pi_{14} = 27.2, \\
\pi_{15} = 17.9, & \pi_{16} = 98.7, & \pi_{17} = 47.9, & \pi_{18} = 27.3, & \pi_{19} = 0.0, \\
\lambda_0 = 64.7, & \lambda_1 = 64.7, & \lambda_2 = 64.3, & \lambda_3 = 52.7, & \lambda_4 = 64.7.
\end{array}$$

Iteration 3. Using these dual values, our algorithm again identifies a positive reduced cost column y_{22} as below:

$$\bar{c}_{22} = 117.0 - (\pi_1 + \pi_6 + \pi_{10}) - \min\{\lambda_2, \lambda_3\} = 11.6.$$

After adding y_{22}, the resulting RMP given in `AvellaRMP3.lp` yields an optimal objective function value of 973.16. The corresponding optimal dual solution is obtained as:

$$\begin{array}{lllll}
\pi_0 = 40.12, & \pi_1 = 24.64, & \pi_2 = 67.04, & \pi_3 = 0.00, & \pi_4 = 0.00, \\
\pi_5 = 14.68, & \pi_6 = 36.08, & \pi_7 = 34.12, & \pi_8 = 44.32, & \pi_9 = 40.12, \\
\pi_{10} = 0.00, & \pi_{11} = 82.28, & \pi_{12} = 0.00, & \pi_{13} = 61.08, & \pi_{14} = 32.92, \\
\pi_{15} = 22.60, & \pi_{16} = 93.08, & \pi_{17} = 52.60, & \pi_{18} = 22.24, & \pi_{19} = 1.48, \\
\lambda_0 = 62.20, & \lambda_1 = 62.20, & \lambda_2 = 60.88, & \lambda_3 = 56.28, & \lambda_4 = 62.20.
\end{array}$$

Iteration 4. Finally, using these dual values, our algorithm finds a positive reduced cost column y_{90} as

$$\bar{c}_{90} = 290.00 - (\pi_7 + \pi_9 + \pi_{16} + \pi_{17}) - \lambda_4 = 7.88.$$

After adding y_{90}, the resulting RMP given in `AvellaRMP4.lp` is solved and no more column with a positive reduced cost is detected by our algorithm. The corresponding optimal objective function value is 974.04. Thus, the proposed column-and-row generation algorithm correctly identifies the optimal solution of the LP relaxation of ITP for this instance.

References

Avella, P., D'Auria, B., and Salerno, S. (2006). A LP-based heuristic for a time-constrained routing problem. *European Journal of Operational Research*, 173:120–124.